

Lines

Questions

Question 1. See the back of this sheet.

Question 2. Find parametric equations for the line contained in the plane $x + y + z = 20$ which also intersects the line $x = y = 2z$ at a right angle.

Question 3. Let L_1 be the line passing through $A(1, -2, 4)$ and $B(2, 1, 3)$, and let L_2 be the line passing through $C(0, 3, -3)$ and $D(2, 4, 1)$.

Are L_1, L_2 parallel, skew, or intersecting? If they intersect, where do they intersect? If not, how far apart are they?

The following are solutions to the problem

“Find the distance d between the point $P(1, -2, 2)$ and the line $\mathbf{r}(t) = \langle 3 + 3t, 2 - t, 5t \rangle$.”

Figure out what is happening in each one.

(1) Solution 1:

$$\begin{aligned} D^2 &= (2 + 3t)^2 + (4 - t)^2 + (5t - 2)^2 \\ &= 35t^2 - 16t + 24 \end{aligned}$$

$$\frac{d}{dt}(D^2) = 70t - 16 = 0$$

$$t = 8/35$$

$$d = D_{\min} = \sqrt{35(8/35)^2 - 16(8/35) + 24} = 2\sqrt{194/35}.$$

(2) Solution 2:

$$3(x - 1) - (y + 2) + 5(z - 2) = 0$$

$$3x - y + 5z - 15 = 0$$

$$3(3 + 3t) - (2 - t) + 5(5t) - 15 = 0$$

$$t = 8/35$$

$$d = \sqrt{(3 + 3(8/35) - 1)^2 + (2 - (8/35) + 2)^2 + (5(8/35) - 2)^2} = 2\sqrt{194/35}.$$

(3) Solution 3:

$$\langle 1, -2, 2 \rangle - \langle 3, 2, 0 \rangle = \langle -2, -4, 2 \rangle$$

$$\langle -2, -4, 2 \rangle \times \langle 3, -1, 5 \rangle = \langle -18, 16, 14 \rangle$$

$$|\langle -18, 16, 14 \rangle| = 2\sqrt{194}$$

$$|\langle 3, -1, 5 \rangle| = \sqrt{35}$$

$$d = 2\sqrt{194/35}.$$

(4) Solution 4:

$$\langle 1, -2, 2 \rangle - \langle 3, 2, 0 \rangle = \langle -2, -4, 2 \rangle$$

$$(\langle 3, -1, 5 \rangle \times \langle -2, -4, 2 \rangle) \times \langle 3, -1, 5 \rangle = \langle 94, 132, -30 \rangle = 2\langle 47, 66, -15 \rangle$$

$$\frac{\langle 47, 66, -15 \rangle \cdot \langle -2, -4, 2 \rangle}{|\langle 47, 66, -15 \rangle|} = -388/\sqrt{6790}$$

$$d = |-388/\sqrt{6790}| = 2\sqrt{194/35}.$$

(5) Solution 5:

$$\langle 2 + 3t, 4 - t, 5t - 2 \rangle \cdot \langle 3, -1, 5 \rangle = 0$$

$$35t - 8 = 0$$

$$t = 8/35$$

$$d = \sqrt{(3 + 3(8/35) - 1)^2 + (2 - (8/35) + 2)^2 + (5(8/35) - 2)^2} = 2\sqrt{194/35}$$

(6) Solution 6:

$$\langle 1, -2, 2 \rangle - \langle 3, 2, 0 \rangle = \langle -2, -4, 2 \rangle$$

$$\frac{\langle 3, -1, 5 \rangle \cdot \langle -2, -4, 2 \rangle}{\langle 3, -1, 5 \rangle \cdot \langle 3, -1, 5 \rangle} \langle 3, -1, 5 \rangle = \frac{8}{35} \langle 3, -1, 5 \rangle$$

$$\langle -2, -4, 2 \rangle - \frac{8}{35} \langle 3, -1, 5 \rangle = \langle -\frac{94}{35}, -\frac{132}{35}, \frac{6}{7} \rangle$$

$$|\langle -\frac{94}{35}, -\frac{132}{35}, \frac{6}{7} \rangle| = 2\sqrt{194/35}.$$